WHY CHILDREN DO NOT DRAW PARALLELS

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After successfully learning to identify pairs of parallel lines, samples of Year 1 and Year 4 students looked for parallels in geometrical figures and copied figures containing parallels. Oblique parallels were usually, but not always, more difficult to find than horizontal and vertical parallels. 62% of the children copied at least one parallel with an error of more than 5 degrees, but most of them judged they had drawn parallels. Asked to check, most realised their error and were able to draw an accurate parallel. The results suggest that young children attempt to preserve parallels in their 2D drawings but are hindered by the complexity of the figure from noticing inaccuracies. Failure to check more carefully might also be a general trend resulting from current teaching methods.

In my keynote address to MERGA-14 in Perth (Mitchelmore, 1991), I reviewed research showing that children often find it difficult to draw parallels, both when making drawings representing 3D figures such as cuboids and when copying straightforward 2D diagrams. The difficulty is particularly pronounced when the parallels are incident to an oblique line; errors are greater for smaller angles of incidence and decrease with the age of the child but do not vanish altogether even among adults.

I discussed some hypotheses which had been put forward to explain this effect:

* children cannot recognise parallels in 2D figures;
* children cannot recognise parallels on 3D objects;
* children cannot draw simple pairs of parallel lines accurately;
* children do not notice parallels on 2D or 3D figures.

All these hypotheses could however be rejected on the basis of research (summarised in Mitchelmore, 1985) showing that children in their first year of primary school can easily recognise and draw simple parallel lines, and that drawing their attention to the parallel lines in a figure does not significantly increase their drawing accuracy; parallel results were obtained for Year 4 students in respect of 3D figures. These findings suggested three guidelines for further research.

1. 2D and 3D drawings must be considered separately: the nature of the drawing task and the distribution of errors are quite different in the two cases.

2. There seems to be a complexity effect. Young children often fail to recognise parallels in figures which contain extra lines, and the drawing error also jumps significantly.

3. Children's errors may be caused by a different concept of what constitutes an "exact copy". Piaget showed that young children often conserve topological rather than Euclidean properties (Piaget & Inhelder, 1956) and Beilin (1984) found that testing of congruence by superposition first arises at about 8 years of age. Perhaps increasing accuracy in drawing parallels is an indicator of a maturing concept of congruence.

The study reported in this paper was designed to investigate whether children's concepts of an "exact copy" of a 2D figure included the preservation of parallels, and how judgements of parallelism were effected by the complexity of the figures.

METHOD

Two groups of 16 children were selected for individual interview, one group from Year 1 (average age 7.1 yr) and one from Year 4 (average age 9.9 yr). Eight children (4 males and 4 females) were selected at random from one class in each year in each of two Catholic schools located in predominantly middle class areas.

As in previous research, parallel lines were called "friends" to avoid possible linguistic problems and interference with verbal knowledge. Using transparent plastic strips, friendly lines were defined as lines which "go along side by side without bumping into each other" however far they go. After four demonstration items, children were asked to identify the friends in 16 pairs of lines at various orientations; they were encouraged to use the plastic strips if they were in any doubt. Errors on the first 8 items were corrected immediately, but no feedback was given on the second 8 items.

Children were then asked if they could see any friendly lines in 12 more complex figures (see Figure 1). Children were prompted ("Are there any more friends?"), if necessary, to ensure that they analysed each figure fully.

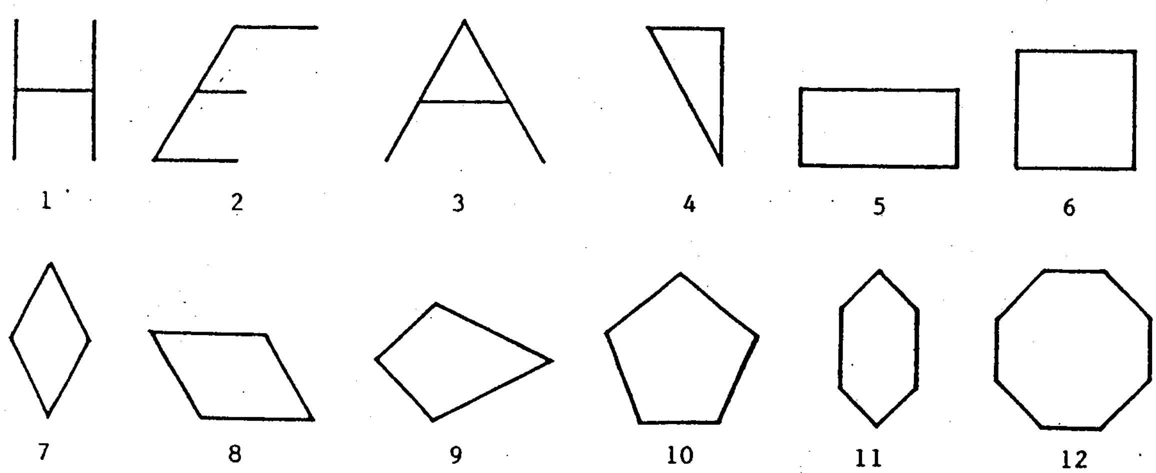

Figure 1: Shapes used for identification of parallels

Children were then given nine drawings to complete. For each item, a diagram containing one red line and one or more black lines was presented on a card. The child had a booklet on which each page showed only the black line(s) and completed the figure by drawing a red line to make it look "exactly like" the given diagram. In three cases children copied parallels, in three cases perpendiculars and in three cases neither; the three drawings in which the target line was parallel to a given line are shown in Figure 2. (The perpendiculars were included as part of a concurrent study of the students' concepts of perpendiculars; see Mitchelmore (1992) for details). Children had already done these copying tasks as a warm-up activity, and now it was suggested to them that looking for friends might make such drawings easier. Before making each drawing, children therefore had to identify any friends in the given diagram; and afterwards, they were asked if there were any friends in their drawing.

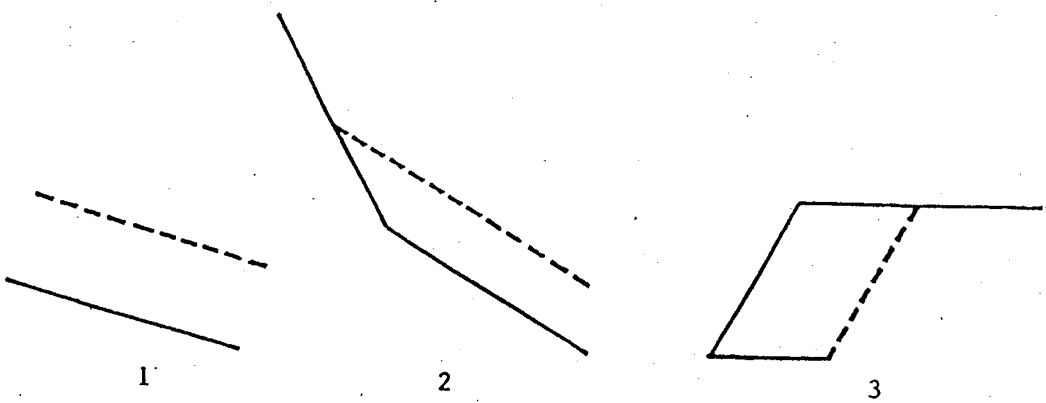

Figure 2: Parallel drawing tasks. The broken lines show the target lines (given in red and copied by the subject).

In a final de-briefing, children were asked if and how identifying friends had helped them do the drawings, whether they had ever thought about friendly lines, whether they could see any friendly lines around them, and whether they had ever studied anything similar in school.

RESULTS

The results confirmed that young school children can easily learn to recognise simple parallels. The Year 1 children made 15 errors on the first 8 figures (a 12% error rate) but only three on the next eight figures (2%); only one Year 4 child made any errors and then only on the first eight figures. In distinction to older students' verbally expressed knowledge of parallels (Mansfield & Happs, 1989), the fact that some pairs of parallel lines were of unequal length did not seem to cause children any difficulties. But occasional comments such as "They would be friendly if they were walking in one direction but not if they were walking in the other direction" (referring to a pair of non-parallel lines) suggest how difficult it might be for children to verbalise the concept of parallelism even when they can recognise parallels without difficulty.

The success rate on the more complex figures (see Figure 1) varied from 94% on Item 1 to 29% on Item 7. In general, the oblique parallels tended to be more difficult to recognise than the horizontal and vertical parallels; in Item 12, for example, the respective success rates were 46% and 85%. Recognising oblique parallels included realising that not all oblique lines are parallel; for example, several children stated that the two lines at the top of Item 11 were parallel.

Table 1: Item difficulties in parallel recognition task (percentages)

Item Group	Item No.	Inclination	Student Group 1	2	3	All
A	1	V	100	100	100	100
	6	HV	100	100	100	100
	11	V	90	100	100	96
	5	HV	82	100	100	93
B	8	H	73	100	100	90
	3	O^-	70	100	100	89
	4	O^-	75	94	100	88
	12	HV	60	100	100	86
	8	O	55	100	100	83
C	2	H	40	73	100	64
	10	O^-	22	75	100	61
	9	O^-	38	63	67	56
	12	O	30	38	100	41
	7	O	27	27	67	31
	11	O	0	38	100	31

* H = horizontal, V = vertical, O = oblique parallel, O^- = oblique non-parallel.

An attempt was made to clarify the sources of difficulty by grouping the various parallels and non-parallels in a manner which would discriminate consistently between children. As a first step, three children (including one from Year 4) who stated that there were no parallels in almost all items were eliminated. Horizontal and vertical parallels were then separated from the obliques, and all item difficulties recalculated. The data were then rearranged in order of item difficulty and student total score, and defensible item and student groupings sought. The result of this rather subjective process was three item groups and three student groups, as shown in Table 1.

Student group 1 consisted of seven Year 1 and four Year 4 students; they were usually successful in Item group A, often wrong in Group B, and usually wrong in Group C. The students in Group 2 (seven from Year 1 and eight from Year 4) were almost always correct in Item groups A and B, but still had difficulties in Group C. The three students in Group 3 (all from Year 4) were successful on almost all items. We note that not all horizontal or vertical parallel items were easier than the oblique items; the presence of oblique lines seemed to make horizontal and vertical parallels more difficult to recognise. Also, differences between Student groups 1 and 2 extended across all orientations.

The parallel in Item 1 (see Figure 2) was generally drawn accurately; only one drawing (made by a Year 1 student during the warm-up) showed an error of more than 5°. On the other hand, the obliques in Items 2 and 3 caused the usual difficulties. In several drawings, the target line started out almost parallel to one line and then curved round to be almost perpendicular to the incident line; in such cases, an "average" straight line was inferred. The numbers of inaccurate drawings of Items 2 and 3 are shown in Table 2. The improvement in accuracy from Year 1 to Year 4 was to be expected, as was the slight difference in difficulty between the items; and the complete absence of any difference in accuracy between the two drawing sessions confirmed that drawing error is not due to lack of attention to parallels in the stimulus figure.

Table 2: Percentage of parallel drawings with an error of more than 5°

	Item 2		Item 3	
Drawing session	Year 1	Year 4	Year 1	Year 4
Warm up	38	12	50	31
After identifying parallels	31	19	50	38

Most interesting in view of the purpose of this study were students' reactions on being asked, in the second session, whether the lines they had drawn were "friends". The great majority, including those whose lines were decidedly inaccurate, claimed their lines were friends (see Table 3). Of the seven accurate drawings not recognised as friends, two were made by a student who did not see any parallels in any of her drawings; in four cases, the parallels were not recognised in the stimulus; and one line was judged as too curved by a student who said "I tried to draw a friend, but it's difficult". Of the five inaccurate drawings judged not to contain friends, two were made by another student who could not see friends in any of his drawings; in two cases, students immediately and spontaneously redrew their lines to be friends; and one student thought the line should be a friend, saw that it was not, but did nothing about it. All in all, the data are strong evidence that the children were expecting to draw friends whenever they saw friends in the given drawing.

Table 3: Percentage of second session drawings claimed to show parallels

	Item 2		Item 3	
Drawing session	Year 1	Year 4	Year 1	Year 4
Error up to 5°	100	80	75	67
Error more than 5°	80	0	63	100

In discussions after all drawings were complete, 30% of the children spontaneously stated that they had copied friends by drawing friends; and all the rest agreed, when prompted, that this is what they had been doing. The 20 children (11 in Year 1 and nine in Year 4) who had made large errors on one or more of Items 2 and 3 in the second session were then asked if their lines really were friends; 75% stated immediately that they were not friends and a further 20% agreed they were not friends after checking with the plastic strips. Of the 19 who now judged that their lines were not friends, 89% indicated that their drawing was not a good one and proceeded to improve it; and of these 17 students, all but one were able (after up to four attempts) to draw an accurate parallel.

Several comments illuminate students' difficulties. They clearly were expecting friends: "It's a friend, but a bit too up", "They look like friends, but slightly off"; and they could see what was wrong: "The line should have been closer here ", "I need to draw it wider". To explain their errors, some students assigned an independent existence to their line: "It wanted to be a friend", "The line turned off a bit"; others blamed themselves: "I didn't know they weren't friends", "You don't really realise it's a bit bent [inaccurate] until you look closer". The task of drawing parallels could seem extraordinarily difficult: "It happened again!" said one student after the third unsuccessful attempt; but most students eventually succeeded: "I stopped now and again and made sure it was straight [parallel]".

No student admitted to having thought about friendly lines before, but all could pick out parallel lines in the interview room. None of the Year 1 students knew the word "parallel" (several had difficulty even saying it) but only one Year 4 student did not know its meaning and most remembered having studied parallels in mathematics lessons.

DISCUSSION

The results of this small investigation confirm many previous findings: parallels can be easily recognised, but they are difficult to draw, and not because children do not pay attention to them. The results also strongly suggest that young children do indeed try to copy parallels and that their idea of an "exact copy" does therefore include parallelism. Many children in this study made this explicit in the interviews, and most of those who did not, acted as if it was this principle that was guiding them; only three out of 32 students showed by their action or comments that they did not believe parallels mattered. Moreover, in contrast to accuracy of recognition and drawing of parallels, and children's experience of parallels in school, there was no discernible difference between Year 1 and Year 4 in terms of the apparent strategies used to copy figures containing parallels. We must reject the hypothesis that children's difficulties in drawing parallels are due to an immature concept of congruence.

A most striking aspect of the interviews was the frequency with which a simple prompt from the interviewer brought about an improvement in children's responses, both in terms of recognition and drawing accuracy. For example, although 20 of the 32 children were apparently content with an inaccurate copy of parallels, after being asked to check their drawings, this number fell to four. (Three of these were from Year 1). However, some children did spontaneously correct unsatisfactory drawings without prompting; there were only two cases of this in Year 1 but eight in Year 4. It seems likely that the real reason why young children often do not draw parallels accurately is that they do not check their drawings carefully enough; and there is some suggestion that the supposed tendency to check one's drawings increases with age.

But why don't children check their drawings? And why should they check their drawings more carefully as they grow older? We can suggest two factors.

Firstly, there were several indications that children had difficulty isolating the various parts of a figure and considering the relation between two lines while ignoring all the other lines. For example, one student had difficulty saying how many lines there were in Items 1 and 2 in Figure 1. Other comments are equally revealing:

* "The two side lines [in Item 1] would be friends if that [middle] line was gone".
* "These lines [the horizontal lines in Item 2] are friends except for that line [on the left]".
* "The lines [in Items 5 and 6] get in each others' way".
* "If they [the opposite sides in Item 5] were apart, they'd be friends".

Also, several students covered up distracting lines when judging friendliness. Such difficulties could explain both why students often did not recognise parallels in complex figures and why they did not check their own drawings carefully enough. The skill of disembedding relevant visual information has a long history of research based on the assumption that it indicates a personality trait known as field independence (Witkin, 1981); it has also been reported as a factor in geometrical performance (Bright, 1975). One thing is clear from this research tradition: disembedding skill improves with age during childhood. One explanation of young children's failure to check their drawings is therefore that checking requires disembedding skills which are still developing. (Development of disembedding skills may also be interpreted within the van Hiele model as one aspect of the transition from the first, global, stage to the second, more analytical but still concrete, stage.)

A second possible explanation is based on social conventions. In primary mathematics classrooms, children may be programmed into believing that the correctness of their work can only be judged by their teachers. The habit of checking one's answers, at least in a mathematical context, may therefore simply not be learnt. (Certainly, secondary school mathematics teachers complain often enough about how difficult it is to get their students to check their answers.) But even if a student checks an answer and sees that it may be incorrect, it requires a degree of courage to admit that to an adult. A sign from an authority figure (e.g. the interviewer) that it is acceptable - or even desirable - to criticise one's own products may bring a barrage of confessions.

The fact that dramatic improvements in drawing accuracy occurred when students were encouraged to check their answers suggests that many students did possess the necessary analytical capability but simply did not bring it into play. The tasks were within their "zone of proximal development" (Vygotsky, 1978). We may surmise that, through the process of making the rationale for their global visual judgements explicit, those students who showed that they could copy parallels more accurately when challenged would in future draw parallels more accurately at the first attempt. Unfortunately, no follow-up drawing tasks were included in the interview protocol to test this conjecture.

IMPLICATIONS FOR TEACHING

Two implications for teaching arise from this study: one narrow and one general. Actually, they are more like reminders than implications.

In narrow terms, the results emphasise that the adult tendency to see geometrical figures as consisting of sides and vertices which can be treated in isolation is something which has to be learnt. Early activities such as making models of common shapes using drinking straws, colouring components (such as opposite sides) of drawn shapes, and finding the number of triangles in a pattern of intersecting lines might enhance the development of such disembedding skills.

More generally, the results remind us of the tremendous gains that can be achieved if students can be taught to check their "answers" (be they the result of a numerical calculation or a geometric construction, or whatever). This cannot be achieved by the teacher repeatedly castigating the students for not checking their answers. Instead, the students must come to feel that there is always a rational basis for deciding whether an answer is correct and that they themselves can make this judgement. That means in practice that, instead of teachers routinely telling students whether their answers are correct, they should expect students to convince themselves that they are correct and occasionally ask them to explain why. In the process of convincing themselves and others, students would learn that mistakes are inevitable and acceptable. (The only non-acceptable mistake is not to realise that one has made a mistake.) This would bring substantial improvements not only in the narrow area of geometrical drawing, but, I believe, in every aspect of mathematics.

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